Problem 6 Solution

$$J_d = en\mu F_x = -en\mu \frac{dV}{dx} = -\mu C'(\underbrace{V_g - V_T}_{V_{gT}} - V)\frac{dV}{dx}$$

$$J_d = -\mu C' (\underbrace{V_{gT} - V}_{\mathcal{U}(x)}) \frac{dV}{dx}$$

$$\mathcal{U}(x) = V_{gT} - V(x)$$

$$\frac{d\mathcal{U}}{dx} = -\frac{dV}{dx}$$

$$\Rightarrow \quad J_d = \mu C' \mathcal{U} \frac{d\mathcal{U}}{dx} = \frac{1}{2}\mu C' \frac{d\mathcal{U}^2}{dx}$$

$$0 = \frac{dJ_d}{dx} = \frac{1}{2}\mu C' \frac{d^2\mathcal{U}^2}{dx^2} \quad \text{or} \quad \frac{d^2\mathcal{U}^2}{dx^2} = 0$$

$$\Rightarrow \quad \mathcal{U}^2(x) = Ax + B$$

BCs:

$$V(0) = 0 \Rightarrow \mathcal{U}^2(0) = V_{gT}^2$$

$$V(L) = V_d \Rightarrow \mathcal{U}^2(L) = (V_{gT} - V_d)^2$$

$$\Rightarrow \quad B = V_{gT}^2 \quad \text{and} \quad AL + V_{gT}^2 = (V_{gT} - V_d)^2$$

$$AL = V_d^2 - 2V_{gT}V_d$$

$$\Rightarrow \quad \mathcal{U}^2(x) = V_d(V_d - 2V_{gT})\frac{x}{L} + V_{gT}^2$$

$$\mathcal{U}(x) = \pm\sqrt{V_d(V_d - 2V_{gT})\frac{x}{L} + V_{gT}^2}$$

$$\mathcal{U}(0) = V_{gT} \qquad \mathcal{U}(L) = V_{gT} - V_d$$

$\Rightarrow$ $\oplus$ solution satisfies BCs.

$$\Rightarrow \quad V(x) = V_{gT} - \mathcal{U}(x) = V_{gT} - \sqrt{V_d\,(V_d - 2V_{gT})\frac{x}{L} + V_{gT}^2}$$

$$F_x(x) = -\frac{dV}{dx} = \frac{V_d\,(V_d - 2V_{gT})/L}{2\sqrt{V_d\,(V_d - 2V_{gT})\frac{x}{L} + V_{gT}^2}}$$